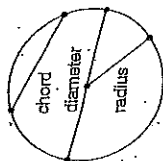


## Exploring Circles

A circle is the set of all points in a plane that are a given distance from a given point in the plane called the center. Various parts of a circle are labeled in the figure at the right.



The distance around a circle is called the **circumference**.

Circumference of a Circle	if a circle has a circumference of $C$ units and a radius of $r$ units, then $C = 2\pi r$ .
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**Example:** Find the circumference of the circle shown at the right.

$$\begin{aligned} C &= 2\pi r \\ C &= 2\pi(13) \\ C &= 26\pi \\ C &\approx 81.7 \end{aligned}$$

The circumference is about 81.7 cm.

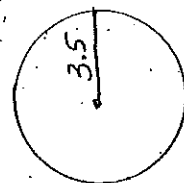
Refer to  $\odot S$  for Exercises 1-6.

1. Name the center of  $\odot S$ .
2. Name three radii of  $\odot S$ .
3. Name a diameter.
4. Name a chord.
5. If  $RT = 8.2$ , find  $SM$ .
6. Is  $\overline{SR} \cong \overline{SM}$ ? Explain.

In Exercises 7-10, the radius, diameter, or circumference of a circle is given. Find the other measures to the nearest tenth.

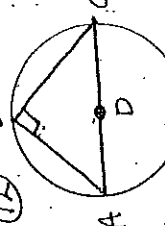
7.  $r = 7$ ,  $d = 14$ ,  $C \approx 44$
8.  $d = 32.4$ ,  $r = 16.2$ ,  $C \approx 103$
9.  $C = 116.5$ ,  $d = 37$ ,  $r = 18.5$
10.  $r = 12$ ,  $d = 24$ ,  $C \approx 75$

Find the exact circumference.



$$C = 7\pi$$

(12) B



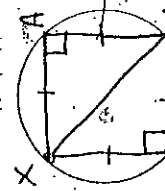
$$\begin{aligned} \text{radius} &= 8 \\ AB &= 8 \\ BC &= ? \end{aligned}$$

(13)



$$\begin{aligned} SA &= 15 \\ AH &= C = ? \end{aligned}$$

(14)



$$AY = XY = ?$$

## Angles and Arcs

An angle whose vertex is at the center of a circle is called a **central angle**. A central angle separates a circle into two arcs called a **major arc** and a **minor arc**. In the circle at the right,  $\angle CEF$  is a central angle. Points C and F and all points of the circle interior to  $\angle CEF$  form a minor arc called arc CF. This is written  $\widehat{CF}$ . Points C and F and all points of the circle exterior to  $\angle CEF$  form a major arc called  $\widehat{CGF}$ .

You can use central angles to find both the degree measure and the length of an arc. The arcs determined by a diameter are called **semicircles** and have measures of 180.

**Examples:** In  $\odot R$ ,  $m\angle ARB = 42$ ,  $RB = 12$ , and  $\widehat{AC}$  is a diameter.

1. Find  $m\widehat{AB}$  and  $m\widehat{ACB}$ .  
Since  $\angle ARB$  is a central angle and  $m\angle ARB = 42$ , then  $m\widehat{AB} = 42$ .  
 $m\widehat{ACB} = 360 - m\widehat{AB} = 360 - 42$  or 318

2. Find the length of  $\widehat{AB}$ .  
First, find what part of the circle is represented by  $\angle ARB$ .

$$\frac{42}{360} = \frac{7}{60}$$

So, the length of  $\widehat{AB}$  is  $\frac{7}{60}$  of the circumference of  $\odot R$ .  
length of  $\widehat{AB} = \frac{7}{60}(2\pi r)$

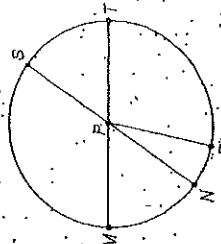
$$= \frac{7}{60}(2\pi)(12) \text{ or about } 8.8 \text{ units}$$

Refer to  $\odot P$  for Exercises 1-8. If  $\widehat{SN}$  and  $\widehat{MT}$  are diameters with  $m\angle SPT = 51$  and  $m\angle NPT = 29$ , determine whether each arc is a minor arc, a major arc, or a semicircle. Then find the degree measure of each arc.

1.  $m\widehat{NR}$
2.  $m\widehat{ST}$
3.  $m\widehat{TSR}$
4.  $m\widehat{MST}$
5.  $\widehat{NR}$
6.  $\widehat{ST}$
7.  $\widehat{TSR}$
8.  $\widehat{MST}$

If  $MT = 15$ , find the length of each arc. Round to the nearest tenth.

9. Find the area of the sector contained  $\angle NPR$



The following theorems state relationships between arcs, chords, and diameters.

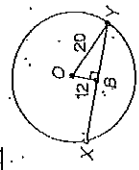
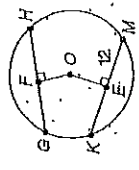
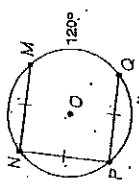
- In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.
- In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.
- In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

**Example:** In the circle,  $O$  is the center,  $OD = 15$ , and  $CD = 24$ . Find  $x$ .

$$\begin{aligned} ED &= \frac{1}{2} CD \\ &= \frac{1}{2} (24) \\ &= 12 \\ (OE)^2 + (ED)^2 &= (OD)^2 \\ x^2 + 12^2 &= 15^2 \\ x^2 + 144 &= 225 \\ x^2 &= 81 \\ x &= 9 \end{aligned}$$

In each circle,  $O$  is the center. Find each measure.

1.  $m\widehat{NP}$
2.  $\widehat{KM}$
3.  $\widehat{XY}$



4. Suppose a chord is 20 inches long and is 24 inches from the center of the circle. Find the length of the radius.

5. Suppose a chord of a circle is 5 inches from the center and is 24 inches long. Find the length of the radius.

6. Suppose the diameter of a circle is 30 centimeters long and a chord is 24 centimeters long. Find the distance between the chord and the center of the circle.

### Inscribed Angles

An inscribed angle of a circle is an angle whose vertex is on the circle and whose sides contain chords of the circle. We say that  $\angle DEF$  intercepts  $\widehat{DF}$ . The following theorems involve inscribed angles.

- If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc.
- If two inscribed angles of a circle or congruent circles intercept congruent arcs or the same arc, then the angles are congruent.
- If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle.
- If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

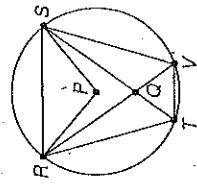
**Example:** In the circle above, find  $m\angle DEF$  if  $m\widehat{DF} = 28$ . Since  $\angle DEF$  is an inscribed angle,  
 $m\angle DEF = \frac{1}{2} m\widehat{DF} = \frac{1}{2} (28)$  or 14.

In  $\odot P$ ,

1. Name the intercepted arc for  $\angle RTS$ .

2. Name an inscribed angle.

3. Name a central angle.



In  $\odot P$ ,  $m\widehat{SV} = 90$  and  $m\angle RPS = 110$ . Find each measure. Also  $m\angle RVT = 48$

4.  $m\angle RVS$
5.  $m\widehat{RS}$
6.  $m\angle RTS$
7.  $m\widehat{TV}$

8.  $m\widehat{RV}$
9.  $m\angle TVQ$